Math 1B Discussion Problems 11 Apr

1. Suppose a population grows according to a logistic model with initial population 1000 and carrying capacity $10,000$. If the population grows to 2500 after one year, what will the population be after another three years?

2. Let c be a positive number. A differential equation of the form $y' = ky^{1+c}$ where k is a positive constant, is called a doomsday equation because the exponent in the expression y^{1+c} is larger than the exponent 1 for natural growth.

   (a) Determine the solution that satisfies the initial condition $y = y_0$.

   (b) Show that there is a finite time $t = T$ (doomsday) such that $\lim_{t\to T} y(t) = \infty$.

3. In a seasonal-growth model, a periodic function of time is introduced to account for seasonal variations in the rate of growth. Such variations could, for example, be caused by seasonal changes in the availability of food. Find the solution of the seasonal growth model $\frac{dP}{dt} = kP\cos(rt), P(0) = P_0$, where k, r are positive constants.